Error Margin for Antenna Gain Measurements
Anatomy of an Error Budget

Vaughn P. Cable
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109-8099

Introduction
The specification of measured antenna gain is incomplete without knowing the error of the measurement. Also, unless gain is measured many times for a single antenna or over many identical antennas, the uncertainty or error in a single measurement is only an estimate. In this paper, we will examine in detail a typical error budget for common antenna gain measurements. We will also compute the gain uncertainty for a specific UHF horn test that was recently performed on the Jet Propulsion Laboratory (JPL) antenna range. The paper concludes with comments on these results and how they compare with the "unofficial" JPL range standard of ± ½ dB."

Measurement Methods
There are two fundamental approaches to measuring antenna gain; 1) by substitution of a known gain standard for the test antenna in the same incident field and noting the ratio of received power for the standard versus the test unit, and 2) by measuring the absolute power transmitted by one antenna and received by another for a given separation and then applying the Friis transmission formula. The latter technique requires two identical test antennas or a third antenna for the case of non-identical test antennas. In this paper, we will review the gain calibration process using two identical UHF horns. The setup for this gain measurement is illustrated in Figure 1. An excellent reference on methods of antenna gain measurement is *Standard Test Procedures for Antennas* [IEEE, 1979].

Gain Equation
Derivation of the error budget for two identical UHF horn antennas starts with the gain based on the Friis transmission formula [S. Ramo, et al., *Fields & Waves in Communications Electronics,* Wiley, 1965, p717], i.e.,

$$\mathbf{G = 4Rf/c[M\,L\,K\,P_r/P_t]^{1/2}}$$

where

R=range between transmit and receive antenna apertures (phase centers)
P_t=power available from the transmit generator
P_r=power delivered to the receive load
L=component losses
f=frequency
$c=3 \times 10^8$ m/s
M=correction for mismatch
$K= K_1K_2K_3K_4K_5K_6$ other factors affecting gain measurement

- K_1= polarization mismatch
- K_2= amplitude taper
- K_3= multipath
- K_4= alignment
- K_5= equipment instabilities
- K_6= mutual coupling

Measurement Error

The error budget in a single gain measurement is derived from estimates of the uncertainties in the variables in the above equation. The relative error in overall gain is obtained by taking the total differential of the above equation and normalizing with respect to total gain, i.e.,

$$\Delta G/G = \Delta R/R + \Delta f/f + \tfrac{1}{2}[\Delta P/P + \Delta M/M + \Delta L/L + \Delta K_1/K_1 + \Delta K_2/K_2 + \Delta K_3/K_3 + \Delta K_4/K_4 + \Delta K_5/K_5 + \Delta K_6/K_6]$$

The signs of the Δ errors are statistically independent and they have a low probability of being the same. Therefore, the gain uncertainty is the RSS of the above quantities, i.e.,

$$\Delta G/G = [(\Delta R/R)^2 + (\Delta f/f)^2 + (\Delta P/2P)^2 + (\Delta M/2M)^2 + (\Delta L/2L)^2 + (\Delta K_1/2K_1)^2 + (\Delta K_2/2K_2)^2 + (\Delta K_3/2K_3)^2 + (\Delta K_4/2K_4)^2 + (\Delta K_5/2K_5)^2 + (\Delta K_6/2K_6)^2]^{1/2}$$

and assuming these errors are 3σ values, this represents the 3σ gain uncertainty.

Mismatch correction

A mismatch correction M must be applied wherever the impedances at each RF interface are not matched, e.g., generator-to-transmit antenna, receive antenna-to-load, etc. In this case, the available power from the source side of an interface is not delivered to the load side of the corresponding interface and the following correction must be applied -

$$M = (1 \pm \rho_S\rho_L)^2 / ((1 - \rho_S^2)(1 - \rho_L^2))$$

where ρ_S and ρ_L are the magnitudes of the voltage reflection coefficients for the source and load, respectively. The $\pm$ accounts for the ambiguity of the sign and the correction M has an uncertainty ΔM that contributes to the uncertainty budget at each frequency.

Freq =	270 MHz	360 MHz	401.5 MHz	437.1 MHz	450 MHz
M_{ave} =	1.15 dB	1.37 dB	.32 dB	.20 dB	.64 dB
$\pm \Delta M$ =	.46 dB	.50 dB	.40 dB	.26 dB	.38 dB

Multipath

The major multipath reflection point was on the ground between the transmit and receive antennas (see Figure 1). Absorber placement was optimized empirically by minimizing cross-pol at the largest separation distance. The longitudinal multipath results at 401 MHz are presented in Figure 2.

Sample Error Budget

Estimates of the significant errors for the two horn gain measurement at 401 MHz are summarized as follows -

Measurement	*Nominal Level*	*Est. Error*
Frequency f	401 MHz	$20\log[\Delta f/f+1] = \pm .21$ dB
Power P_r/P_t	10 dB	$10\log[\Delta(P_r/P_t)+1] = \pm .05$ dB
Distance R	15.24 m	$20\log[\Delta R/R+1] = \pm .006$ dB
Impedance mismatch correction M	.32 dB	$10\log[\Delta M/M+1] = \pm .4$ dB
Loss L	19.8 dB	$10\log[\Delta L/L+1] = \pm .05$ dB
Polarization mismatch $\Delta K_1/K_1$	0 dB	$10\log[\Delta K_1/K_1+1] = \pm .25$ dB
Aperture taper $\Delta K_2/K_2$	0 dB	$10\log[\Delta K_2/K_2+1] = \pm .005$ dB
Multipath $\Delta K_3/K_3$	0 dB	$10\log[\Delta K_3/K_3+1] = \pm .10$ dB
Alignment $\Delta K_4/K_4$		*Negligible*
Equipment drift $\Delta K_5/K_5$	0 dB	$10\log[\Delta K_4/K_4+1] = \pm .25$ dB
Mutual coupling $\Delta K_6/K_6$		*Negligible*

For this case, the gain error bar at 401 MHz in dB is given by –

$$\mathbf{ERROR = 10\log[\Delta G/G+1] = \pm .29\ dB}$$

The final results for the 270 – 450 MHz frequency band are present in Figure 3. This figure also shows results for the same antenna measurement made in 1997 and the agreement is within the error bars of this latest measurement.

Conclusions

As we have seen, the development of an accurate error budget requires estimates from a wide variety of sources, e.g., equipment specifications, range geometry, user readings, etc. Some of the errors are rigorous in the sense that they can be derived from equations (M). Others are back–of-the-envelope order-of-magnitude calculations (aperture taper) or measured quantities (multipath, drift). The negligible errors in the above budget (mutual coupling, alignment) were deemed insignificant based on simple calculations. A rigorous error budget is dependent on the antenna engineer's expectations of what looks right about the measurement setup and the data, and what does not. The error result computed here is fairly close to the unofficial guess expressed at the outset. However, this is no accident. In the end, it's the engineer's experience with the range and engineering judgment that provide insight into identifying and controlling the sources of measurement error and their probable levels.

Acknowledgment

This work was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

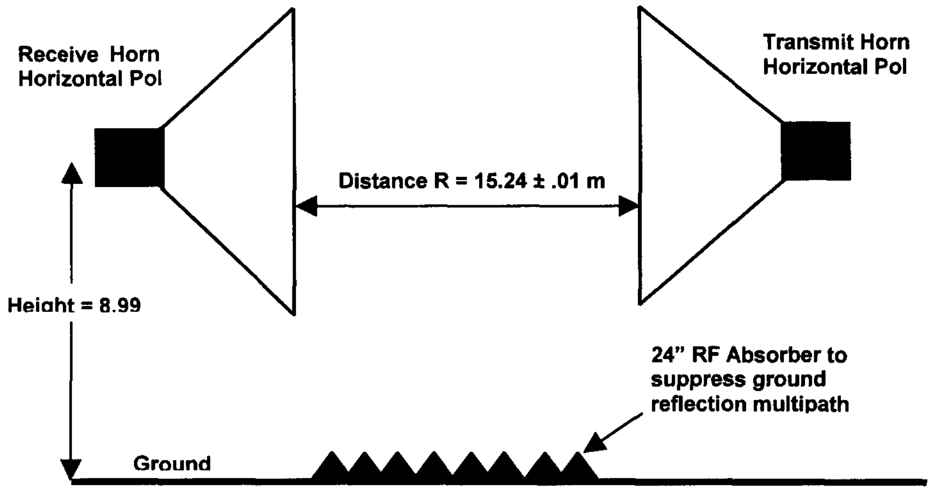

Figure 1. UHF horn measurement setup.

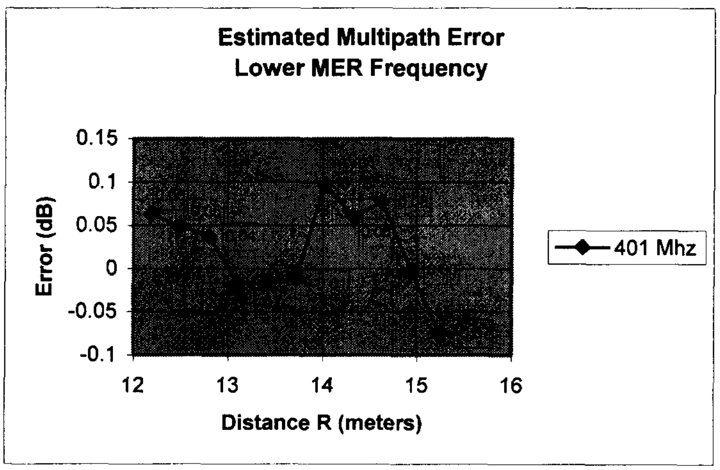

Figure 2. Multipath measurement ($1/R^2$ roll-off with distance has been subtracted).

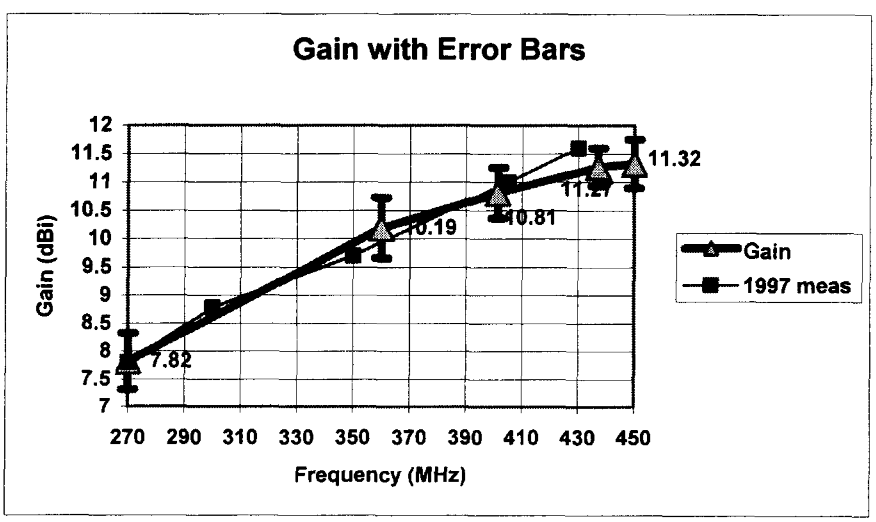

Figure 3. Measured Gain of UHF Horn

Conclusion:
From the results of our investigation it was found that with a few straight segments it was not possible to produce circular polarization with a low value of cross polarization over a wide angular region. If the number of segments is increased to a large value the antenna shape would approach that of a spiral or curl antenna [5]. Additional results on the crooked wire and other antennas will be presented in the symposium.

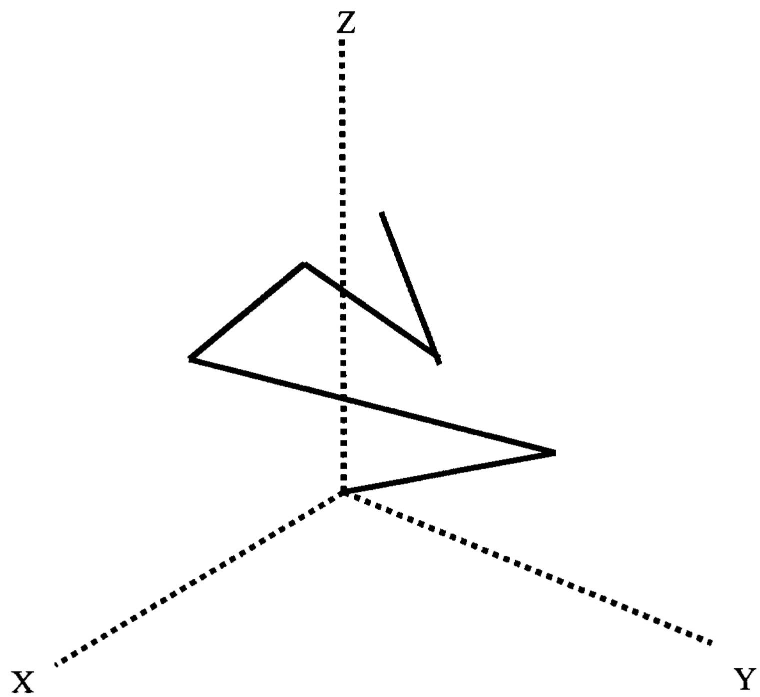

Fig. 1 A sketch of the crooked wire antenna

Acknowledgement
We wish to acknowledge many helpful discussions we had with Dr. John Huang at Jet Propulsion Laboratory. The research described in this work was carried out at Jet Propulsion Laboratory, California Institute of Technology under a contract with the National Aeronautics and Space Administration.

References:
[1] E.E. Altshuler and D.S. Linden, "Design of wire antennas using genetic algorithms," in Electromagnetic Optimization by Genetic Algorithms, Edited by Y. Rahmat-Samii and E. Michielssen, pp. 211-248, John Wiley & Sons, Inc., 1999.
[2] E.E. Altshuler, "Design of a vehicular antenna for GPS/IRIDIUM using a Genetic Algorithm," IEEE Transactions on Antennas and Propagation, vol. 48, pp. 968-972, June 2000.